INSTITUTE OF MATHEMATICS AND APPLICATION

BHUBANESWAR

ENTRANCE TEST FOR ADMISSION INTO B.Sc. (HONOURS) IN MATHEMATICS AND COMPUTING,2024-2025

Maximum Marks:100 **Time: 2 hours**

INSTRUCTION TO CANDIDATES

- Ensure that this Test Booklet contains 08 printed pages with multiple choice questions.

- Candidates are required to check that the Test Booklet does not have any discrepancy(ies) link unprinted or torn or missing pages,missing questions etc.If so, get it replaced by a complete test booklet before attempting to answer. No extra time will given, if replaced afterwords.
- Each of the questions/incomplete statements is followed by four options/choices marked as (a),(b),(c),(d) under each question/statement,of which only one of them is correct/most appropriate.
- For each question, mark the correct/most appropriate option/choice by putting a cross ($\times$) mark in the appropriate box of the answer sheet provided to you. In case, a candidate feels that there is multiple correct options/choices, the candidate has to mark the option/choice which he/she feels is the most appropriate/best. In any case,only one option/choice has to be marked for each question.More than one option/choice marked in the answer sheet against a question number will be deemed as incorrect.
- If you mark your option/choice at any place other than the box provided, it will not be evaluated.
- Each correct answer carries 2 marks.
- Use of any written/ printed material, calculator, docu-pen, any communication devices like cell phones/i-pads etc,inside the examination hall is not allowed. Candidates found with such items will be reported and his/her candidature will be summarily cancelled.
- Blank sheet(s) for doing rough work/calculations is/are appended at the end of the Test Booklet.

  Warning: Any malpractice or any attempt to commit any kind of malpractice in the examination hall will disqualify the candidate.

MULTIPLE - CHOICE QUESTIONS

Throught this booklet, $\mathbb{R}$ stands for the set of real numbers and z stands for complex numbers

1. If R is a relation defined on a set $A = \{a, b, c\}$ then relation $R = \phi$ is

(a) equivalence relation
(b) symmetric, transitive but not reflexive
(c) symmetric but neither reflexive nor transitive
(d) none of these.

2. If $f(x) = \max(x, 1/x)$ for $x > 0$ where $\max(a, b)$ denotes the greater of the two real number a and b then for $0 < k < 1$ the value of $f(k)f(1/k)$ is

(a) $1/k$ (b) k (c) $1/k^2$ (d) $1/k^3$.

3. The fundamental period of $f(x) = sin^4 x + cos^4 x$ is

(a) π (b) $3\pi/4$ (c) $\pi/4$ (d) $\pi/2$.

4. If y is a function of x defined by $3^{x+y} = 3^x + 3^y$ then the domain of $y(x)$ is

(a) $(1, \infty)$ (b) $(3, \infty)$ (c) $(0, \infty)$ (d) $(-1, \infty)$.

5. Let f and g be the functions defined by $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{x}{1-x}$.Then $(fog)^{-1}(x)$ is

(a) $\frac{1}{x}$ (b) x (c) $\frac{1}{x+1}$ (d) $x + 1$.

6. If $f(x+2) = \frac{1}{2}\{f(x+1) + \frac{4}{f(x)}\}$ and $f(x) > 0$ for all $x \in \mathbb{R}$ then $\lim_{x\to\infty} f(x)$ is

(a) 1 (b) 3 (c) -2 (d) 2 .

7. If $f(x) = \sum_{k=1}^{n}(x - \frac{1}{k})(x - \frac{1}{k+1})$ then $\lim_{n\to\infty} f(0)$ is

(a) 0 (b) -1 (c) 1 (d) 2 .

8. If the graph of the function $y = f(x)$ is symmetric about the line $x = 2$ then

(a) $f(x+2) = f(x-2)$ (b) $f(x) = f(-x)$

(c) $f(2+x) = f(2-x)$ (d) $f(x+1) = f(1-x)$.

9. If $f(x) = 27x^3 + \frac{1}{x^3}$ and α, β are the roots of $3x + \frac{1}{x} = 2$ then

(a) $f(\alpha) = 10$ (b) $f(\beta) = 12$ (c) $f(\alpha) = f(\beta) = -10$ (d) $f(\alpha) \neq f(\beta)$.

10. If $5f(x) + 3f(\frac{1}{x}) = x + 2$ and $y = xf(x)$ then $(\frac{dy}{dx})_{x=1}$ is equal to

(a) 8/7 (b) 1 (c) 7/8 (d) 1/7 ,

11. The number of divisors of 1029, 1547 and 122 are in

(a) AP (b) GP (c) HP (d) none of these.

12. If for all x, y the function f defined by $f(x) + f(y) + f(x).f(y) = 1$ and $f(x) > 0$ then

(a) $f'(x)$ does not exists (b) $f'(x) = 0$ for all x

(c) $f'(0) < f'(1)$ (d) none of these.

13. If $\lim_{x \to \infty} (1 + \frac{\alpha}{x} + \frac{\beta}{x^2}) = e^2$ then

(a) $\alpha = 1, \beta = 2$ (b) $\alpha = 2, \beta = 1$

(c) $\alpha = 1, \beta$ is any real constant (d) $\alpha = -1, \beta = 1$.

14. Let $f(x) = \int_0^x t sin\frac{1}{t}dt$. Then the number of points of discontinuity of function $f(x)$ in the open interval $(0, \pi)$ is

(a) 0 (b) 1 (c) 2 (d) infinite.

15. The equation of the tangent to the curve $y = e^{-|x|}$ at the point where the curve cuts the line $x = 1$ is

(a) $x + y = e$ (b) $e(x + y) = 1$ (c) $y + ex = 2$ (d) $x + ey = 2$.

16. A ballon is pumped at the rate of a cm^3/min. The rate of increase of its surface area when the radius is b cm is

(a) $\frac{2a^2}{b^4} cm^2/min$ (b) $\frac{a}{2b} cm^2/min$ (c) $\frac{2a}{b} cm^2/min$ (d) none of these.

17. If x be a number which exceeds its square by the greatest possible quantity then x is equal to

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5.

18. Let $g(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ for all values of x. Then

(a) $g(x)$ is increasing when $f(x)$ increasing (b) $g(x)$ increasing when $f'(x) < 0$

(c) $g(x)$ decreasing when $f'(x) > 0$ (d) none of these.

19. If $f(x)$ is a polynomial of degree 4 with $f(2) = -1, f'(2) = 0, f''(2) = 2, f'''(2) = -12$ and $f^{(iv)}(2) = 24$ then the value of $f''(1)$ is

(a) 20 (b) 22 (c) 24 (d) 26.

20. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are continuous functions then the value of the integral

$\int\limits_{-\pi/2}^{\pi/2} (f(x) + f(-x))(g(x) - g(-x))dx$ is

(a) π (b) $\pi/2$ (c) 1 (d) 0.

21. The value of $\int\limits_{2}^{5} \frac{f(x)}{f(x)+f(7-x)}dx$ is

(a) 3/2 (b) 1/2 (c) 5/2 (d) none of these.

22. The area enclosed by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is

(a) 1/2 sq unit (b) 1/3 sq unit (c) 4/3 sq unit (d) 5/3 sq unit.

23. If $f(2a - x) = f(x)$ and $\int\limits_{0}^{a} f(x)dx = k$ then $\int\limits_{0}^{2a} f(x)dx$ is

(a k (b) $2k$ (c) $3k$ (d) $k/2$.

24. Let $f(x)$ be a differentiable function and $f(1) = 2$. If $\lim\limits_{x\to 1} \int\limits_{2}^{f(x)} \frac{2t}{x-1}dt = 4$ then the value of $f'(1)$ is

(a) 1 (b) 2 (c) 3 (d) 4.

25. If $f(1/x) + x^2 f(x) = 0, x > 0$ and $I = \int\limits_{1/x}^{x} f(t)dt, 1/2 \leq x \leq 2$, then I is equal to

(a) $f(2) - f(1/2)$ (b) 0 (c) $f(1/2) - f(2)$ (d) none of these.

26. Let $f(x) = (sinx)^{\frac{1}{\pi - 2x}}, \quad x \neq \pi/2$. If $f(x)$ is continuous at $x = \pi/2$ then $f(\pi/2)$ is

(a) e (b) 1 (c) 0 (d) none of these.

27. The values of a for which the equation $sin^4 x + cos^4 x = a$ has real solution

(a) $2 \leq a \leq 3$ (b) $1/4 < a < 1/2$ (c) $1/2 \leq a \leq 1$ (d) none of these.

28. If $1 < x < \sqrt{2}$, then the number of solution of the equation $tan^{-1}(x-1) + tan^{-1}x + tan^{-1}(x+1) = tan^{-1}3x$ is

(a) 1 (b) 2 (c) 3 (d) 0.

29. If α, β, γ are the altitudes of a triangle ABC and $2s$ denotes it perimeter and Δ is its area then $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is equal to

(a) $\frac{\Delta}{s}$ (b) $\frac{s}{\Delta}$ (c) $s.\Delta$ (d) none of these.

30. if $f(x) = \begin{vmatrix} a^2 + x & ab & ac \\ ab & b^2 + x & bc \\ ac & bc & c^2 + x \end{vmatrix}$ then $f(x)$ is divisible by

(a) x^2 (b) x^3 (c) x^4 (d) none of these.

31. If $log_{\frac{1}{2}} \frac{|z|^2 + 2|z| + 4}{2|z|^2 + 1} < 0$ then the region traced by z is

(a) $|z| > 1$ (b) $|z| > 3$ (c) $|z| < 3$ (d) $|z| > 4$.

32. If $z^4 + z^3 + 2z^2 + z + 1 = 0$ then $|z|$ is equal to

(a) 1 (b) 2 (c) 1/2 (d) 3.

33. If $z\overline{(z+\alpha)}+\overline{z}(z+\alpha)=0$, where α is a complex constant,then z is represented by a point on

(a) circle (b) parabola (c) ellipse (d) hyperbola.

34. Let $z=\frac{cos\theta+isin\theta}{cos\theta-isin\theta}$, $\pi/4<\theta<\pi/2$. Then $\arg z$ is

(a) $\pi+2\theta$ (b) $2\theta-\pi$ (c) 2θ (d) none of these

35. The number of ways in which a mixed double game can be arranged from among 9 married couples if no husband and wife play in the same game is

(a) 1020 (b) 1252 (c) 1352 (d) 1552.

36. The number of integral solution of $x+y+z+t=29$ when $x\geq 1, y\geq 2, z\geq 3$ and $t\geq 0$ is

(a) 2500 (b) 2600 (c) 2700 (d) 2800.

37. The number of integers between 1 and 1000000 have sum of the digits equal to 18 is

(a) 25925 (b) 25926 (c) 25927 (d) 25928.

38. In a club election the number of contestants is one more than the number of maximum candidates for which a voter can vote. If the total number of ways in which a voter can vote be 62 then the number of candidates is

(a) 5 (b) 6 (c) 7 (d) 8.

39. If a variable line remains at a constant distance $3p$ from origin and meets the coordinate axes at A and B then the locus of the centroid of the triangle AOB is

(a) $x^{-2}+y^{-2}=p^{-2}$ (b) $x^{-2}+y^{-2}=2p^{-2}$ (c) $x^{-2}+y^{-2}=3p^{-2}$ (d) $x^{-2}+y^{-2}=4p^{-2}$.

40. If the chord $y=mx+1$ of the circle $x^2+y^2=1$ subtends an angle of measure $\pi/4$ at the major segment of the circle then the value of m is

(a) 1 (b) 2 (c) 3 (d) none of these.

41. The locus of the middle point of a chords of a parabola which subtend a right angle at the vertex of the parabola $y^2 = 4ax$ is

(a) a circle (b) an ellipse (c) a parabola (d) a hyperbola.

42. The image of a point (1,3,4) with respect to the plane mirror $2x - y + z + 3 = 0$ is

(a) (1,3,2) (b) (-3,5,2) (c) (3,5,-2) (d) (3,-5,2).

43. The number of vectors of unit length perpendicular to the vector $\overrightarrow{a} = (1, 1, 0)$ and $\overrightarrow{b} = (0, 1, 1)$ is

(a) 1 (b) 2 (c) 3 (d) 4.

44. The volume of the tetrahedron whose vertices are given by the vectors $-\overrightarrow{i} + \overrightarrow{j} + \overrightarrow{k}, \overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$ and $\overrightarrow{i} + \overrightarrow{j} - \overrightarrow{k}$ with reference to the fourth vertex as origin is

(a) 1/3 (b) 2/3 (c) 4/3 (d) 5/3.

45. If two dice are thrown then the probability that the sum of the numbers coming up on them is 9 when it is known that number 5 always occurs on the first die is

(a) 1/4 (b) 1/5 (c) 1/6 (d) 1/7.

46. A five digit number is formed by using the digits 0,1,2,3,4 and 5. The probability that the number is divisible by 6 is

(a) 3/50 (b) 7/50 (c) 9/50 (d) 11/50.

47. The remainder when 2^{2003} is divisible by 17 is

(a) 1 (b) 2 (c) 8 (d) 9.

48. The coefficient of x^{10} in the expansion of $(1 + x^2 - x^3)^8$ is

(a) 475 (b) 476 (c) 575 (d)576.

49. If $f(x) = \begin{vmatrix} 1 & sinx & 1-cosx \\ 1+sinx & cosx & 1+sinx-cosx \\ sinx & sinx & 1 \end{vmatrix}$ then $\int\limits_0^{\pi/2} f(x)dx$ is equal to

(a) 1/2 (b) 1/3 (c) -1/2 (d) -1/3.

50. The value of $\begin{vmatrix} x & x^2-yz & 1 \\ y & y^2-zx & 1 \\ z & z^2-xy & 1 \end{vmatrix}$ is

(a) 1 (b) -1 (c) 0 (d) $-xyz$.

***** THE END *****