Institute of Mathematics and Applications, Bhubaneswar

Entrance Test for Admission into M.A./M.Sc. Courses

(Mathematics with Data Science / Computational Finance)

2024-2025

Maximum Marks: 100 Time: 2 Hours.

INSTRUCTIONS

1. This question booklet contains 6 pages (including this page) with 50 multiple choice questions.

2.Each of the questions/incomplete statements is followed by four choices marked as (A), (B),(C), and(D) below each question of which only one of them is correct.

3. Answer each question by writing exactly one choice A or B or C or D in the space provided for it in the answer sheet, which is supplied separately. This question paper is meant to be retained by you.

4. More than one choice marked against a question will be deemed as incorrect answer.

5. Each correct answer carries 2 marks and incorrect or no answer carries 0 mark.

6. Use of calculator, log table, mobile phone, or any electronic gadget is not allowed.

QUESTIONS

1. If 1,a,b, and c are roots of the equation $x^4 = 1$,what is the value of (1-a)(1-b)(1-c) ?

(A) 0 (B) 4 (C) 1 (D) None of these

2. In how many ways can n-couples sit around a table so that men and women alternate?

(A) (2n-1)! (B) (n!)(n!) (C) ((n-1)!)(n-1)! (D) ((n-1)!)n!

3. In how many ways can 15 chocolates of same type be distributed among 4 children so that each child gets at least 5

(A) 120 (B)210 (C) 35 (D) None of these

4. A coin and a die are tossed simultaneously. What is the probability of getting a tail and a number greater than 3?

(A)1/2 (B) 1/3 (C) 1/4 (D) None of these

5. S is a set containing 10 elements. What is the number of binary operations that can be defined on S?

(A) 100^{10} (B) 100 (C) 10^{100} (D) None of these

6.What should be the domain of the function f(x)=|sin x| so that it has an inverse?

(A) [0,π/2] (B) [-π/2 , π/2] (C)[0 , π] (D) None of these

7.What is the area of the region bounded by $y=x^2$, and y=x?

(A)1/2 (B) 1/3 (C) 1/6 (D) None of these

8.If $+_n$,and x_n denote addition modulo n and multiplication modulo n respectively, then what is the value of

$\{5\ x_6\ (3 +_8 7)\}\ x_5\ 4$?

(A) 4 (B)3 (C)1 (D) None of these

9. Which of the following statements is false?

(A) A group order 11 is cyclic (B) A group order 111 is cyclic

(C) A group order 1111 is cyclic (D)None of these

10.What is the rank of the matrix $\begin{pmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{pmatrix}$?

(A) 0 (B) 1 (C)2 (D)3.

11. The eigenvectors corresponding to distinct eigenvalues of a matrix are

(A) Orthogonal (B) Linearly dependent (C)Linearly independent (D)None of these.

12. The order of a proper subgroup of a group G of order 31 is

(A) 1 (B) 31 (C) 1 or 3 (D)None of these.

13. Which of the following is not correct?

(A) Every cyclic group is an abelian group.

(B)Every group of odd order is cyclic.

(C)The order of a cyclic group and the order of its generating elements is equal.

(D)Every subgroup of a cyclic group is cyclic.

14. If p is a prime and a is any number prime to p , then p divides

(A) $a^p - 1$ (B) a^{p-1} (C) $a^{p-1} - 1$ (D) $a^p + 1$

15. Which one of the following results is incorrect?

(A) $\Delta x^n = nx^{n-1}$ (B)) $\Delta x^{n-1} = nx^{n-1}$ (C)) $\Delta^n e^x = e^x$ (D) $\Delta cosx = -sinx.$

16.Which of the following set of vectors form a basis for R^3 ?

(A) {(-1,0,0),(1,1,1),(1,2,3)} (B) { (0,1,2),(1,1,1),(1,2,3)}

(C) { (0,0,0), (-1,1,0),(2,0,0)} (D) { (0,0,1),(1,0,1),(1,0,0) }

17. What is the dimension of the space V= {(x,y,z,w) ε R^4 | x=w,y=2z} ?

(A) 4 (B) 3 (C)2 (D) None of these.

18. Suppose T is a linear transformation from R^2 into R such that the kernel of T is {(x,-x):x ε R}.If T(1,0) = 1, then the value of T(1,1) is

(A) 0 (B) 1 (C) 2 (C) None of these.

19. Given, $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ the maximum number of possible basic solutions is

(A) 3 (B) 4 (C) 2 (D) 6.

20.Choose Maclaurin's series expansion of e^x from the following alternatives.

(A) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\cdots\cdots \quad \forall x \in R.$

(B) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\cdots\cdots \quad \forall x \in R.$

(C) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots\cdots \quad \forall x \in R.$

(D) None of the above.

21. If a real number $c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfies the conclusion of Rolle's theorem for the function f(x)=cos x in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ then what is the value of c?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) π (D) 0

22. If some c ϵ (2 , 4) satisfies the conclusion of Lagrange's mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ in [2, 4] then what is the value of c?

(A) 12 (B) 6 (C) $\sqrt{2}$ (D) $\sqrt{6}$.

23. What is the set of all real numbers x at which |sin x| is not differentiable ?

(A) { nπ | n is an integer} (B) { 2nπ|n is an integer}

(C){ nπ/2 | n is an integer} (D) None of these

24. For what range of value(s) of p the function f, defined by

$f(x)=x^p \sin(1/x)$, for $x \neq 0$; f(0) =0, is continuously differentiable at x =0 ?

(A) p≥1 (B) 1<p≤2 (C) p>2 (D) None of these

25.The interior points of the open interval (0,1) with respect to the complex plane is

(A) (0,1) (B) {0,1} (C){0} (D) empty set

26. If H_1 and H_2 are two subgroups of G , then which of the following is also a subgroup of G?

(A) $H_1 \cap H_2$ (B) $H_1 U H_2$ (C) $H_1 H_2$ (D)None of these.

27. In the group { 0,1,2,3} with addition modulo operation, the order of 2 is

(A) one (B) two (C) three (D)four.

28. If f is a homomorphism from (Z,+) to (R^+ , *) , and f(2) = 1/2 , then the value of f(6) is

(here + denotes addition on the set of integers ,and * denotes multiplication on reals)

(A) 1/8 (B) 3/2 (C) 8 (D) 6

29. Let H={ 5n| n is an integer} .What is the total number of left cosets of H in the group Z(the set of integers)

(A) 15 (B) 10 (C) 5 (D) 1

30.$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ has the solution

(A) $y = c_1e^{-2x} + c_2e^{x}$ (B) ce^{-2x} (C) $y = c_1e^{-2x} + c_2e^{-x} + c_3$ (D) None of these.

31. The integrating factor for the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ is

(A) 1/(x+1)   (B) x+1   (C) $1/(x^2+1)$ (D) x^2+1.

32. The differential equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 50e^{2x}$ has particular integral

(A) $(2e^{2x})/3$ (B) $2e^{2x}$ (C) e^{2x} (D) None of these.

33. The particular solution to the initial value problem
$y'' - 2y' + y = 0$, $y(0)=1$, $y'(0)=-1$ is

(A) $(1+2x)e^x$ (B) $(1-2x)e^x$

(C) $(2x-1)e^x$ (D) None of these

34. Let $f(x)=x^2-5$ for x real. If $\{x_n\}$ is a sequence of iterates defined by the Newton-Raphson method to approximate a solution of f(x)=0 with $x_1=0$, then the value of x_2 is

(A) 1.5   (B) 2.7   (C) 3   (D) None of these

35. What is the greatest rate of increase of the scalar function f(x,y,z)=xyz at the point (1,0,3)?

(A) 1   (B) 2   (C) 3   (D) None of these

36. If a sphere with center (2,-3,6) touches the YZ-plane, then what is its radius?

(A) 6   (B) 3   (C) 2   (D) None of these

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37. The probability that a teacher will give an unannounced test during any class is 1/5 . If a student is absent twice, then the probability that he misses at least one test is

(A) 2/5   (B) 4/5   (C) 7/25   (D) 9/25.

38. Given the probability density function $f(x) = \begin{cases} e^{-x}, x \geq 0 \\ 0, x < 0 \end{cases}$ then the value of Cumulative Distribution function at x=2 is

(A) $1+e^{-2}$ (B) e^{-2} (C) e^{-1} (D) $1-e^{-2}$.

39. If $F(x) = \int_1^x \frac{dt}{(t^2+1)^2}$, then F'(1)=........

(A) 0   (B) -1/2   (C) 1/4   (D) None of these.

40. What is the point of maximum or minimum of the function $f(x,y)=0.5x^2 + xy + 2y^2 - 4x - 4y + y^3$?

(A) (4,0)   (B) (3,1)   (C) (0,0)   (D) Does not exist.

41. $\int_0^1 \frac{dx}{x^{\frac{2}{3}}}$

(A) diverges to $+\infty$ (B) diverges to $-\infty$ (C) is equal to 0   (D) None of these.

42. If we change the order of integration, $\int_0^1 (\int_0^{1-y} f(x\,,y)dx)dy$ becomes
(A) $\int_0^{1-y} (\int_0^1 f(x\,,y)dx)dy$ (B) $\int_0^{1-y} (\int_0^1 f(x\,,y)dy)dx$ (C) $\int_0^1 (\int_0^{1-x} f(x\,,y)dx)dy$ (D) $\int_0^1 (\int_0^{1-x} f(x\,,y)dy)dx.$

43.If $f(\,x\,,y) = \begin{cases} \frac{x-y}{x+y}\,, for\ x \neq -y \\ 1\,, for\ x = -y \end{cases}$ then as (x, y) → (0 , 0), f (x , y) approaches
(A) 1 (B) -1 (C) 0 (D) no limit.

44.The function f is continuous on [0 , 1] such that f(0)= -2 , f(1/2)= ½ , and f (1)= -1, we can then conclude that
(A) f is non-zero on [0 , 1] (B) f vanishes at least twice on [0 , 1]
(C)) f vanishes exactly twice on [0 , 1] (D)) f vanishes exactly once on [0 , 1].

45. A sequence { x_n } is monotonic and bounded ,then
(A) all subsequences of { x_n } converge to the same limit.
(B) there exist a subsequence that diverges
(C) there exist at least two subsequences { x_n } which converge to distinct points
(D)None of these.

46. The least upper bound of the set $\{\frac{n+1}{n}\,, n \in N\,\}$ is
(A) 2 (B) 1 (C) 0 (D) non existent

47. The series $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha-1}}$ is convergent if
(A) $\alpha > 1$ (B)) $\alpha \leq 1$ (C)) $\alpha > 2$ (D)) $\alpha \leq 2.$

48. The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\cdots$
A) converges to e (B) converges to ln(1/2) (C) diverges (D) converges to ln2.

49. Which of the following statements is false?
(A) a function which is differentiable at a point is continuous at that point.
(B) a function continuous on an interval [a , b] is integrable on [a , b]
(C) a continuous function having countable number of discontinuities on [a , b] is integrable.
(D) a function continuous on [a , b] is differentiable on [a , b].

50.which one of the following statements for the graph of $y=1/(x-3)^2 -4$ is false?
(A) The graph is symmetrical about the X-axis
(B) The graph has a vertical asymptote whose equation is x=3
(C) The graph has a horizontal asymptote whose equation is y=-4
(D) The X-intercepts are 7/2 and 5/2.

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