INSTITUTE OF MATHEMATICS AND APPLICATION

BHUBANESWAR

ENTRANCE TEST FOR ADMISSION INTO B.Sc. (HONOURS) IN MATHEMATICS AND COMPUTING,2023-2024

/laximum Marks:100 **Time: 2 hours**

INSTRUCTION TO CANDIDATES

- Ensure that this Test Booklet contains 08 printed pages with multiple choice questions.
- Candidates are required to check that the Test Booklet does not have any discrepancy(ies) link unprinted or torn or missing pages,missing questions etc.If so, get it replaced by a complete test booklet before attempting to answer. No extra time will given, if replaced afterwords.
- Each of the questions/incomplete statements is followed by four options/choices marked as (a),(b),(c),(d) under each question/statement,of which only one of them is correct/most appropriate.
- For each question, mark the correct/most appropriate option/choice by putting a cross ($\times$) mark in the appropriate box of the answer sheet provided to you. In case, a candidate feels that there is multiple correct options/choices, the candidate has to mark the option/choice which he/she feels is the most appropriate/best. In any case,only one option/choice has to be marked for each question.More than one option/choice marked in the answer sheet against a question number will be deemed as incorrect.
- If you mark your option/choice at any place other than the box provided, it will not be evaluated.
- Each correct answer carries 2 marks.
- Use of any written/ printed material, calculator, docu-pen, any communication devices like cell phones/i-pads etc,inside the examination hall is not allowed. Candidates found with such items will be reported and his/her candidature will be summarily cancelled.
- Blank sheet(s) for doing rough work/calculations is/are appended at the end of the Test Booklet.

Warning: Any malpractice or any attempt to commit any kind of malpractice in the examination hall will disqualify the candidate.

MULTIPLE - CHOICE QUESTIONS

'hrought this booklet, $\mathbb{R}$ stands for the set of real numbers and z stands for complex numbers

1. If $f(x)$is a polynomial satisfying $f(x).f(1/x) = f(x) + f(1/x)$ and $f(3) = 28$, then $f(4)$ is

(a) 63 (b) 65 (c) 67 (d) 68.

2. If $f(x) = \frac{4^x}{4^x+2}$ and $f(x) + f(1-x) = 1$ then the value of $f(1/1997) + f(2/1997) + \ldots + f(1996/1997)$is

(a) 996 (b) 997 (c) 998 (d) 999.

3. Let $f : \mathbb{R} \to \mathbb{R}$. If for some positive constant k such that $f(x+k) = 1+[2-3f(x)+3(f(x))^2-(f(x))^3]^{1/3}$ then $f(x)$ is

(a) Periodic with period 3k (b) Periodic with 4k (c) Periodic with 2k (d) Not periodic.

4. If $f(x)$ is a polynomial satisfying $f(x)f(y) = f(x) + f(y) + f(xy) - 2$ for all real x and y and $f(2) = 5$ then $\lim_{x \to 3} f(x) =$

(a) 9 (b) 10 (c) 35 (d) None of these.

5. If $f(x), g(x)$ be differentiable functions and $f(1) = g(1) = 2$ then $\lim_{x \to 1} \frac{f(1)g(x)-f(x)g(1)-f(1)+g(1)}{g(x)-f(x)}$ is

(a) 0 (b) 1 (c) 2 (d) 3.

6. If α is a repeated roof of $ax^2 + bx + c = 0$ then the $\lim_{x \to \alpha} \frac{sin(ax^2+bx+c)}{(x-\alpha)^2}$ is

(a) 0 (b) a (c) b (d) c.

7. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$ then

(a) f is continuous but not differentiable at $x = 0$
(b) f is differentiable at $x = 0$
(c) f is not differentiable at $x = 0$
(d) f is not continuous at $x = 0$

8. Let $f(\frac{x+y}{2}) = \frac{f(x)+f(y)}{2}$ for all real x and y. If $f'(0) = -1$ and $f(0) = 1$ then $f(2)$ is

(a) 1 (b) -1 (c) 2 (d) -2.

9. If one of roots of $x^2+f(a)x+a=0$ is equal to the third power of other for all real a then $f(x)$ is equal to

(a) $x^{1/3}+x^{4/3}$ (b) $x^{1/4}(1+x^{1/2})$ (c) $x^{1/3}(1+x^{4/3})$ (d) $-x^{1/4}(1+x^{1/2})$.

10. Let $f(x+y)=f(x)f(y)$ for all x and y. If $f(5)=2$ and $f'(0)=3$ then $f'(5)$ is equal to

(a) 4 (b) 5 (c) 6 (d) None of these.

11. If $f'(x)=g(x)$ and $g'(x)=-f(x)$ for all x and $f(2)=4=f'(2)$ then $f^2(19)+g^2(19)$ is

(a) 8 (b) 16 (c) 32 (d) 64.

12. If $log_{1/2}\frac{|z|^2+2|z|+4}{2|z|^2+1}<0$ then the region traced by z is

(a) $|z|<3$ (b) $1<|z|<3$ (c) $|z|>1$ (d) $|z|<2$.

13. If $|z-2+i|\leq 2$ then the greatest value of $|z|$ is

(a) $\sqrt{5}-2$ (b) $\sqrt{5}+2$ (c) $\sqrt{5}+3$ (d) $\sqrt{5}-3$.

14. If $arg(z^{1/3})=\frac{1}{2}arg(z^2+\bar{z}z^{1/3})$ then the value of $|z|$ is

(a) 4 (b) 0 (c) 1 (d) 2 .

15. If $z=1-t+i\sqrt{t^2+t+2}$, where t is real parameter then the locus of z is

(a) a hyperbola (b) an ellipse (c) a straight line (d) a circle.

16. Let $f(x)=\lim_{n\to\infty} n(x^{1/n}-1), x>0$ such that $f(1/x)=kf(x)$ then the value of k is

(a) 1 (b) -1 (c) 3 (d) -3.

17. If $f:\mathbb{R}\to\mathbb{R}$ by $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$ for all $x\in\mathbb{R}$ then the value of $f(2)$ is

(a) 1 (b) -1 (c) 2 (d) -2.

18. The set of values of a for which the function $f(x) = (\frac{\sqrt{a+4}}{1-a} - 1)x^5 - 3x + log5$ decreases for all x is

(a) $(-\infty, \infty)$ (b) $[1, \infty)$ (c) $(-\infty, 2)$ (d) None of these.

19. If $f(x-y), f(x).f(y), f(x+y)$ are in A.P for all x, y and $f(0) \neq 0$ then

(a) $f(1)+f(-1) = 1$ (b) $f'(2)+f'(-2) = 0$ (c) $f'(1)+f'(-1) = 0$ (d) None of these.

20. Let $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf(x) + f'(x)$. Then

(a) $f'(1) + f'(2) = 0$ (b) $f'(1) + g'(1) = 0$ (c) $g'(4) + f'(4) = 1$ (d) None of these.

21. If $5f(x) + 3f(1/x) = x + 2$ and $y = xf(x)$ then the value of $\frac{dy}{dx}$ at $x = 1$ is

(a) 8 (b) 5/3 (c) 7/8 (d) 1.

22. The point on the curve $x^{3/2} + y^{3/2} = a^{3/2}$, where the tangent is equally inclined to the axes is

(a) $(a/2, a/3)$ (b) $(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{3}})$ (c) $(\frac{a}{2^{2/3}}, \frac{a}{2^{2/3}})$ (d) None of these.

23. If $f(0) = f'(0) = 0$ and $f''(x) = tan^2x$ then $f(x)$ is

(a) $logcosx + \frac{1}{2}x^2$ (b) $logsecx + \frac{1}{2}x^2$ (c) $logsecx - \frac{1}{2}x^2$ (d) None of these.

24. If $f(x)$ is a quadratic polynomial such that $f(0) = 2, f'(0) = -3$ and $f''(0) = 4$ then $\int_{-1}^{1} f(x)dx$ is equal to

(a) 2 (b) -2 (c) 3 (d) -3.

25. If $\int_{-2}^{3} f(x)dx = 5$ and $\int_{1}^{3}(2 - f(x))dx = 6$ then the value of $\int_{-2}^{1} f(x)dx$ is

(a) 3 (b) -3 (c) 7 (d) -7.

26. $f(x) = \int_{x}^{x^2} \frac{dt}{1+t^3}$ then $f'(2)$ is equal to

(a) 28/585 (b) -28/585 (c) -29/585 (d) 29/585.

27. Four couples (husband and wife) decide to form a committee of four members. The number of different committees that can be formed in which no couple find a place is

(a) 8 (b) 10 (c) 16 (d) 20.

28. The number of ways of selecting 10 balls out of an unlimited number of white,red, blue and green balls is

(a) 86 (b) 84 (c) 270 (d) 286.

29. In an examination of 9 papers a candidate has to pass in more papers than the number of papers in which he fails in order to be sucessesful.The number of ways in which he can be sucessesful is

(a) 255 (b) 193 (c) 256 (d) 310.

30. How many integral solution are there to $x + y + z + t = 29$ when $x \geq 1, y \geq 1, z \geq 1$ and $t \geq 1$?

(a) 1600 (b) 2600 (c) 3600 (d) None of these.

31. In how many ways can three persons, each throwing a single die once,make a sum of 15 ?

(a) 8 (b) 20 (c) 10 (d) 16.

32. The number of zeros at the end of 100! is

(a) 18 (b) 20 (c) 24 (d) 30.

33. A variable chord is drawn through origin to the circle $x^2 + y^2 - 2ax = 0$. The locus of the cente of the circle drawn on this chord as diameter is

(a) $x^2 + y^2 + ax = 0$ (b) $x^2 + y^2 + ay = 0$ (c) $x^2 + y^2 - ax = 0$ (d) $x^2 + y^2 - ay = 0$.

34. The equation to the line which touches the rectangular hyperbola $9x^2 - 9y^2 = 8$ and parabola $y^2 = 32x$ is

(a) $9x + 3y - 8 = 0$ (b) $9x + 3y + 8 = 0$ (c) $3x + 9y + 8 = 0$ (d) $3x + 9y - 8 = 0$.

35. The equation of the image of the pair of rays $y = |x|$ about the line $x = 1$ is

(a) $|y| = x + 2$ (b) $|y| + 2 = x$ (c) $y = |x - 2|$ (d) None of these.

36. The length of the common chord of the parabola $2y^2 = 3(x + 1)$ and the circle $x^2 + y^2 + 2x = 0$ is

(a) $2\sqrt{3}$ (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{4}$.

37. The vector $\vec{i}(\vec{a} \times \vec{i}) + \vec{j} \times (\vec{a} \times \vec{j}) + \vec{k} \times (\vec{a} \times \vec{k})$ is equal to

(a) $\vec{0}$ (b) $\vec{a}$ (c) $2\vec{a}$ (d) None of these.

38. The projection of the vector $\vec{i} + \vec{j} + \vec{k}$ on the line whose vector equation is $\vec{r} = (3 + t)\vec{i} + (2t - 1)\vec{j} + 3t\vec{k}$, t being the scalar parameter is

(a) $\frac{1}{\sqrt{14}}$ (b) $\frac{5}{\sqrt{14}}$ (c) $\frac{6}{\sqrt{14}}$ (d) $\frac{7}{\sqrt{14}}$.

39. Let ABC be an equilateral triangle whose orthocenter is the origin O. If $\vec{OA} = \vec{a}, \vec{OB} = \vec{b}$ then $\vec{OC}$ is

(a) $\vec{a} + \vec{b}$ (b) $\frac{\vec{a}+\vec{b}}{2}$ (c) $-(\vec{a} + \vec{b})$ (d) $-2(\vec{a} + \vec{b})$.

40. Three dice are thrown. The probability of getting a sum which is a perfect square is

(a) 2/5 (b) 9/20 (c) 1/4 (d) 17/108.

41. A bag contains 14 balls of two colours,the number of balls of each colour being the same. 7 balls are drawn at random one by one.The ball in hand is returned to the bag before each new draw. If the probability that at least 3 balls of each colour are drawn is p then

(a) $p > 1/2$ (b) $p = 1/2$ (c) $p < 1/3$ (d) $p < 1/2$.

42. The Probability of a number n showing in a throw of a dice marked 1 to 6 is proportional to n. Then the probability of the number 3 showing in a throw is

(a) 1/2 (b) 1/6 (c) 1/7 (d) 1/21.

43. If a, b and c are all different from zero, and $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ is equal to zero, then the value of $1/a + 1/b + 1/c$ is

(a) abc (b) $a^{-1}b^{-1}c^{-1}$ (c) $-a-b-c$ (d) -1.

44. The value of a for which the system of equations:
$a^3x + (a+1)^3y + (a+2)^3z = 0,\ ax + (a+1)y + (a+2)z = 0,\quad x + y + z = 0$
has a non zero solution is

(a) 1 (b) 0 (c) -1 (d) None of these.

45. Let $f(x) = \begin{vmatrix} cosx & x & 1 \\ 2sinx & x^2 & 2x \\ tanx & x & 1 \end{vmatrix}$. Then $\lim_{x \to 1} \frac{f(x)}{x}$ is

(a) 2 (b) -2 (c) 1 (d) -1.

46. The number of real solution of equation $cos^7x + sin^5x = 1, 0 \leq x \leq 2\pi$ is

(a) 1 (b) 2 (c) 3 (d) 4.

47. The sum of $cot^{-1}3 + cot^{-1}7 + cot^{-1}13 + cot^{-1}21 + cot^{-1}31$ is

(a) $cot^{-1}\frac{5}{7}$ (b) $cot^{-1}\frac{7}{5}$ (c) $\pi/4$ (d) $\pi/2$.

48. If $I = \int_{1/\pi}^{\pi} \frac{1}{x} sin(x - 1/x)dx$ then I is equal to

(a) π (b) $\pi - \frac{1}{\pi}$ (c) 0 (d) 2π.

49. Let $A = (3,4), B = (1,2)$ and $P = (2k-1, 2k+1)$ is a variable point such that $PA + PB$ is minimum. Then k is

(a) 8/7 (b) 7/9 (c) 7/8 (d) 0.

50. Let a variable plane remains at a constant distance $3p$ from origin meets the cordinate axes at A, B and C. Then the locus of the centroid of the triangle ΔABC is

(a) $1/x^2 + 1/y^2 + 1/z^2 = 3/p^2$ (b) $1/x^2 + 1/y^2 + 1/z^2 = 2p^2$ (c) $1/x^2 + 1/y^2 + 1/z^2 = 1/p^2$

(d) None of these

***** THE END *****