INSTITUTE OF MATHEMATICS AND APPLICATIONS, BHUBANESWAR

ENTRANCE TEST FOR ADMISSION INTO

M.A./M.Sc. in Mathematics with Data Science, 2023-2024
M.A./M.Sc. in Computational Finance, 2023-24

Maximum Marks: 100 Time: 2 hours

INSTRUCTION TO CANDIDATES

- Ensure that this Test Booklet contains 06 printed pages with multiple choice questions.
- Candidates are required to check that the Test Booklet does not have any discrepancy (ies) link unprinted or torn or missing pages, missing questions etc. If so, get it replaced by a complete test booklet before attempting to answer. No extra time will be given, if replaced afterwards.
- Each of the questions/incomplete statements is followed by four options/choices marked as (a),(b),(c),(d) under each question/statement of which only one of them is correct/most appropriate. • For each question, mark the correct/most appropriate option/choice by putting a cross (×) mark in the appropriate box of the answer sheet provided to you. In case, a candidate feels that there is multiple correct options/choices, the candidate has to mark the option/choice which he/she feels is the most appropriate/best. In any case, only one option/choice has to be marked for each question. More than one option/choice marked in the answer sheet against a question number will be deemed as incorrect.
- If you mark your option/choice at any place other than the box provided, it will not be evaluated.
- Each correct answer carries 2 marks.
- Use of any written/ printed material, calculator, docu-pen, any communication devices like cell phones/i-pads etc, inside the examination hall is not allowed. Candidates found with such items will be reported and his/her candidature will be summarily cancelled.
- Blank sheet(s) for doing rough work/calculations is/are appended at the end of the Test Booklet. Warning:

Any malpractice or any attempt to commit any kind of malpractice in the examination hall will disqualify the candidate.

(MULTIPLE - CHOICE QUESTIONS)

1. If A and B are real symmetric matrices of order nxn, then which of the following is true?

A. AA^T B. $A = A^{-1}$ C. $AB = BA$ D. $(AB)^T = BA$

2. If 1, 2 ,and 3 are the diagonal elements of the diagonal matrix A , then what are the eigen values of the inverse of A?

A. 1,2,3 B. 1, 1/2 , 1/3 C. 0,1/2 ,1/3 D. Not possible to determine.

3. Let H={ ... , -6,-3,0,3,6, ... }.What is the total number of left cosets of H in the group Z(the set of integers) with respect to addition.

A. 1 B. 4 C. 3 D. Infinite

4. Let f: A→B is a mapping from set A to set B, and f is one –one and onto. Then f^{-1} :B→A is:

A. One-one and onto B. One-one but not onto C. Not one-one but onto D. None of these.

5. The order of a proper subgroup of a group G of order 17 is :

A. 1 B. 17 C. 1 & 3 D. None of these.

6. Which of the following is not correct?
   A. Every cyclic group is an abelian group
   B. Every group of odd order is cyclic
   C. The order of a cyclic group and the order of its generating elements is equal
   D. Every subgroup of a cyclic group is cyclic.

7. If p is a prime and a is any number prime to p , then p divides :

A. $a^p - 1$ B. a^{p-1} C. $a^{p-1} - 1$ D. $a^p + 1$

8. Let $\sum a_n$ is a series of complex numbers which converges absolutely then every rearrangement of $\sum a_n$
   A. Converges
   B. Diverges
   C. Converges and converges to the same sum
   D. None of these

9. $L^{-1}\left\{\frac{1}{s^2+9}\right\} =$

A. $\frac{1}{9}sin3t$ B. $\frac{1}{3}cost$ C. $\frac{1}{3}sin3t$ D. $\frac{1}{3}cos3t$

10. The order of convergence of secant method is

A. 2 B. 1.6 C. 1 D. None of these.

11. Which one of the following result is incorrect ?

A. $\Delta x^n = nx^{n-1}$ B. $\Delta x^{n-1} = nx^{n-1}$ C. $\Delta^n e^x = e^x$ D. $\Delta cosx = -sinx$

12. The convex combination of two points x,yϵ X is referred as :
(A)$(1-\lambda)x + \lambda y, 0 \leq \lambda \leq 1$
(B) $(1-\lambda)x + \lambda y$, λϵ R
(C) $\lambda x + \lambda y, 0 \leq \lambda \leq 1$
(D)$\lambda x + \lambda y$, λϵ R

13. Suppose f:[a , b] →R is continuous on [a , b] and f is differentiable on (a , b) . If f(a)=f(b), there is cϵ (a , b):f/(c)=0. This is the statement of

(A) Rolle's theorem (B) Mean value theorem
(C) Intermediate value theorem (D) None of these.

14. Choose Maclaurin's series expansion of sinx from the following alternatives.
(A) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\cdots\cdots \ \forall x \in R.$
(B) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\cdots\cdots \ \forall x \in R.$
(C) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots\cdots \ \forall x \in R.$
(D) None of the above.

15. If the function $f(x) = \sqrt{x^2 - 4}$ in [2, 4] satisfies the Lagrange's mean value theorem , then there exists some cϵ(2 , 4) that satisfies the conclusion of the theorem. What is the value of c?
A. 12 B. 6 C. $\sqrt{2}$ D. $\sqrt{6}$

16. If a function f(z) is analytic at a point z_0 , then which of the following statement is false ?
(A) f is differentiable at z_0
(B) f is not continuous at z_0
(C) f is defined at z_0
(D) f is continuous at z_0

17. The value of m so that $2x - x^2 + my^2$may be harmonic is

A. 0 B. 1 C. 2 D. 3

18. The radius convergence of the power series $f(z) = \sum \frac{n+1}{(n+2)(n+3)} z^n$ is

A. 1 B. 2 C. 3 D. 4

19. The function f(z)= cosz
(A) is analytic every where
(B) has singularity at $z = \pm \frac{(n+1)\pi}{2}$
(C) has singularity at $z = \frac{n\pi}{2}$
(D) None of these

20. For Euler Φ-function Φ(6) is equal to

A. 3 B. 2 C. 4 D. 6

21. The unit digit of 3^{101} is

A. 1 B. 2 C. 3 D. 6

22. If G= { 0,1,2,3,4,+5} , the order of 2 is

A. one B. two C. three D. Four

23. The order of identity element in a group G is

A. one B. zero C. order of group D. None of these

24. If f is a homomorphism from (Z,+) to (R^+ , *) and if f(2) = 1/3 ,f(3)=2 , then the value of f(7) is

A. 1/9 B. 4 C. 1/3 D. 2/9

25. A ring (R, +,.) is said to have zero divisor , if

(A) $a, b \in R , a.b = 0 \Rightarrow a \neq 0 \text{ or } b \neq 0$

(B) $a, b \in R , a.b = 0 \Rightarrow a \neq 0 \text{ and } b \neq 0$

(C) $a, b \in R , a.b = 0 \Rightarrow a = 0 \text{ or } b = 0$

(D) $a, b \in R , a.b = 0 \Rightarrow a = 0 \text{ and } b = 0$

26. A: F is a field B: F is integral domain

A. A ⇒B B. B ⇒ A C. A⇔B D. None of these.

27. f is said to be a homomorphism of R into R' , if

(A) f(a-b)= f(a)- f(b) +f(ab),∀a,b ϵ R

(B) f(a+b)= f(a)+ f(b) ,∀a,b ϵ R

(C) f(ab)= f(a) f(b) ,∀a,b ϵ R

(D) both (B) and (C)

28. The solution of ordinary differential equation of n order contains

(A) n-arbitrary constants

(B) more than n-arbitrary constants

(C) no arbitrary constants

(D) None of these

29. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ has the solution

(A) $y = c_1e^{-2x} + c_2e^x$

(B) ce^{-2x}

(C) $y = c_1e^{-2x} + c_2e^{-x} + c_3$

(D) None of these

30. The necessary condition for the equation M (x , y) dx + N (x , y)dy =0 to be exact is

A. $\frac{\partial N}{\partial y} = \frac{\partial M}{\partial x}$ B. $\frac{\partial N}{\partial y} = -\frac{\partial M}{\partial x}$ C. $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$ D. $\frac{\partial N}{\partial x} = -\frac{\partial M}{\partial y}$

31. For $\frac{d^2y}{dx^2} + 4y = tan2x$ solving by variation of parameters. The value of wronskian W is

A. 1 B. 2 C. 3 D. 4

32. The differential equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 50e^{2x}$ have particular integral

A. $(2e^{2x})/3$ B. $2e^{2x}$ C. e^{2x} D. None of these.

33. In a party, persons of the same sex hug and opposite sex shake hands. If there were 15 hand shakes in the party how many hugs were there ?

A. 15 B. 13 C. 8 D. Cannot be found from the given data.

34. In the binomial distribution the variance σ^2 and mean μ are related by

A. $\sigma^2 = q\mu$ B. $\sigma^2 = \mu/q$ C. $q^2\sigma^2 = \mu$ D. None of these.

35. Given the probability density function $f(x) = \begin{cases} e^{-x}, x \geq 0 \\ 0, x < 0 \end{cases}$ then the value of the cumulative distribution function at x=3 is

A. $1+ e^{-3}$ B. e^{-3} C. e^{-1} D. $1- e^{-3}$

36. What is the maximum degree of the polynomial for which Simpson's 1/3 rule is exact ?

A. 1 B. 2 C. 3 D. 4

37. If $f(x, y) = \begin{cases} \frac{x-y}{x+y}, for\ x \neq -y \\ 1, for\ x = -y \end{cases}$ then as $(x, y) \rightarrow (0, 0)$, $f(x, y)$ approaches

A. 1 B. -1 C. 0 D. no limit.

38. For $\epsilon = 1$ the maximum value of δ which satisy the requirement of $\epsilon - \delta$ definition of the limit for $\lim_{x\to 0}(2x + 1)$ is

A. 1/4 B. 1/2 C. 3/4 D. 1

39. Let $x \in R$ .Then $|3x - 1|$ is differentiable

A. on R B. on R-{0} C. on R-{1/3} D. on R- {-1/3}

40. The least upper bound of the set $\{\frac{n+1}{n}, n \in N\}$ is

A. 2 B. 1 C. 0 D. non existant

41. The sequence $\{x_n\}$ where $x_n = (-1)^n logx$ is

A. diverges to $+\infty$ B. diverges to $-\infty$ C. oscillate D. Converges

42. The sequence $\sqrt{n^2+n}-n$

A. diverges to $+\infty$ B. diverges to $-\infty$ C. converges to 1 D. converges to ½

43. The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\cdots$

A. converges to e
B. converges to ln(1/2)
C. diverges
D. converges to ln2

44. The set S= { 7+1 , 7+½ , 7+1/3 , ...} $\subset R$ is

(A) closed (B) open
(C) neither open nor closed (D) Both open and closed

45. Which of the following statements is false?

(A) arbitrary union of open sets is open
(B) arbitrary intersection of closed sets is closed
(C) arbitrary union of closed sets is closed
(D) finite intersection of open sets is open

46. Which of the following statements is false?

(A) a function which is differentiable at a point is continuous at that point.
(B) a function continuous on an interval [a , b] is integrable on [a , b]
(C) a continuous function having countable number of discontinuities in [a , b] is integrable.
(D) a function continuous on [a , b] is differentiable on [a , b]

47. Which of the following statements is false ?

(A) a set is a closed if it contains all its limit points
(B) a set is closed if its complement is open
(C) S is closed if S= $\bar{S}$
(D) one of the above statement is false

48. One hundred cards are numbered from 1 to 100. The probability that a randomly chosen card has a number divisible by 4 and 6 is

A. 0.41 B. 0.08 C. 0.04 D. 0.02

49. If two friends are born in the month of June , then the probability that both of them have the same birth day is

A. 30/365 B. 2/30 C. 1/30 D. None of these.

50. The expectation of the number on a die when thrown is

A. 3 B. 4 C. 3.5 D. None of these

***