

M.A./M.Sc. Scholarship Test-2016
Subject: Mathematics
Answer all questions from each section.

Time: 2 Hours

Max. Marks: 100

Section-A

(Multiple Choice Questions: Q.1-Q.10)

Each question carries 2 marks each.

Pick out the most appropriate qualifying statement from the multiple choices. None of the statements or one statement may qualify. Write none, if none of the statements qualify, or write down the labels of the qualifying statement (amongst (A), (B), (C), and (D)) in your answer script.

1. Let $\{a, b, c\}$ be a basis of the vector space \mathbb{R}^3 over the field \mathbb{R} . Consider the following statements:

$S_1 : \{a + b + c, a + 2b - c, a - 3c\}$ is a basis of \mathbb{R}^3 .

$S_2 : \{a + b, b + c, a - c\}$ is a basis of \mathbb{R}^3 .

Then which one of the following is true ?

(A) Both S_1 and S_2 . (B) Only S_1 . (C) Only S_2 . (D) Neither S_1 nor S_2 .

2. For the sequence $\{x_n\}_{n \geq 1}$, defined (recursively) by $x_1 = 1$, and $x_{n+1} = \frac{1}{2} \left(x_n + \frac{3}{x_n} \right)$, $n \geq 1$, which one of the following is false ?

(A) For all $n \geq 1, x_n \geq 3$.

(B) The sequence $\{x_n\}_{n \geq 1}$ is monotonically decreasing.

(C) The sequence $\{x_n\}_{n \geq 1}$ converges to 3.

(D) All the above statements are false.

3. If S_5 denote the symmetric group of all permutations of the five symbols 1, 2, 3, 4, 5, then the highest possible order of an element in this group is ...

(A) 3. (B) 5. (C) 6. (D) None of these.

4. For the function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined by

$$f(x) = \begin{cases} |x|^\alpha \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases} \quad (\alpha \in \mathbb{R}),$$

which one of the following is false ?

(A) f is continuous at 0, if and only if $\alpha > 0$.

(B) f is differentiable at 0 if and only if $\alpha > 1$.

(C) f is continuously differential at 0, if and only if $\alpha > 2$.

(D) All the above statements are false.

5. Consider the following statements:

$S_1 : A$ complex valued function $f = u + iv$ is analytic in a domain $G \subseteq \mathbb{C}$.

$S_2 : f = u + iv$ is such that $u_x = v_y, u_y = -v_x$ in G .

Then which one of the following is correct ?

(A) S_1 and S_2 are equivalent statements.

(B) S_1 does not imply S_2 .

(C) S_2 implies S_1 .

(D) S_2 is necessary condition for S_1 , but not sufficient in general.

6. For the function $f(x, y) = \begin{cases} xy \tan\left(\frac{y}{x}\right), & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$, which one of the following is correct ?
- (A) $xf_x + yf_y + xy = 2f$. (B) $yf_x + xf_y = 2f$.
 (C) $xf_x + yf_y = 2f$. (D) $xf_x - yf_y + xy = 2f$.
7. If $L^{-1}\left(\frac{1}{s}\right) = 1$ and $L^{-1}\left(\frac{1}{s^2}\right) = t$, then what is the value of $L^{-1}\left(\frac{1}{s^3}\right)$?
 (Here L^{-1} denotes the inverse Laplace transform).
- (A) $\frac{t^2}{2}$. (B) $\frac{1}{2t^2}$. (C) $-\frac{t^2}{2}$. (D) $-\frac{1}{2t^2}$.
8. Which one of the following is false ?
- (A) An Abelian group of order 6 is cyclic. (B) A group of order 35 is cyclic.
 (C) A group of order 9 is abelian. (D) All the above statements are false.
9. The Newton-Raphson iterative method for finding the value of \sqrt{N} ($N > 0$) is
- (A) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$. (B) $x_{n+1} = \frac{1}{2}(x_n - Nx_n)$.
 (C) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$. (D) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{\sqrt{N}}{x_n}\right)$.
10. If $y_1(x) = x^{-1}$ is one of the solutions of the differential equation: $2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 3y = 0$, then the other linearly independent solution is
- (A) x^{-2} . (B) \sqrt{x} . (C) x . (D) x^2 .

Section-B

(Objective Type Questions: Q.11-Q.30.)

Each question carries 2 marks.

Answer each question in one word or group of words or at most in one sentence.

11. Let the circle $\gamma := \{z \in \mathbb{C} : |z| = 3/2\}$ be oriented in the counter-clockwise direction, and $I = \int_{\gamma} z^7 \cos\left(\frac{1}{z^2}\right) dz$. Then the value of I equals to
12. What is the value of $\int_0^5 \max\{x^2, 6x - 8\} dx$?
13. What is the area of the region $f(D)$, where D is the unit disk in the plane \mathbb{R}^2 , and the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is given by $f(x, y) = \left(\frac{x-y}{\sqrt{2}}, \frac{y-x}{\sqrt{2}}\right)$?
14. If the x -coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4, then what is the z -coordinate of the point ?
15. Let $T_1, T_2: \mathbb{R}^5 \rightarrow \mathbb{R}^3$ be the linear transformations such that $\text{rank}(T_1) = \text{rank}(T_2) = 3$. Then, what is the rank of the linear transformation $T_3: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, defined by $T_3 \circ T_1 = T_2$?
16. If $y_1(t) = e^t \sin t$ and $y_2(t) = e^t \cos t$ are the two solutions of a second order differential equation, then find the value of their Wronskian $W(y_1(t), y_2(t))$.

17. What is the domain of convergence of the power series: $\sum_{n=1}^{\infty} \frac{(3x-1)^{2n}}{n3^n}$ ($x \in \mathbb{R}$) ?
18. Find all the asymptotes parallel to co-ordinate axes of the curve: $y = \frac{x}{(x^2-1)}$.
19. If f is continuously differentiable real-valued function defined in the open interval $(-1, 4)$ such that $f(3) = 5$ and $f'(x) \geq -1$ for all $x > -1$, then what is the maximum possible value of $f(0)$?
20. If z is a complex variable and \bar{z} denotes the complex conjugate of z , then find the value of $\lim_{z \rightarrow 0} \frac{(\bar{z})^2}{z^2}$.
21. The value(s) of $\alpha \in \mathbb{Z}_3$ for which $\frac{\mathbb{Z}_3[x]}{\langle x^2 + \alpha x + 1 \rangle}$ is a field is ...
22. Suppose $f : G_1 \rightarrow G_2$ is an onto homomorphism of groups. If the order of the group G_1 is 125, and the order of the $\text{Ker}(f)$ is 5, then what is order of the group G_2 ?
23. Suppose a point P is chosen randomly inside the unit circle in \mathbb{R}^2 . What is the probability that the point P is closer to the center than the circumference of the circle ?
24. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, and $r = |\vec{r}|$, then what is the value of $\text{div} \left(\frac{\vec{r}}{r^3} \right)$?
25. If $u(x, t)$ is the solution of the wave equation: $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ with $u(x, 0) = \cos(5\pi x)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$, then what is the value of $u(1, 1)$?
26. If \vec{a} and \vec{b} are irrotational vectors, then is it true that $\vec{a} \times \vec{b}$ an irrotational vector ? Why ?
27. What is/are the interior point(s) of the open interval $(-1, 1)$ with respect to \mathbb{C} ?
28. For what value(s) of p , is the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$ convergent ?
29. If P_n denotes the Legendre polynomial of degree n , then what is the value of $\int_{-1}^1 x^{100} P_{101}(x) dx$?
30. Write down the condition to be satisfied by the real numbers a, b, c and d in order that the sphere $x^2 + y^2 + z^2 = 1$ and the plane $ax + by + cz + d = 0$ have a non-empty intersection.

Section-C

(Short Answer Type Questions: Q.31-Q.38)

Each question carries 5 marks.

Show your calculations.

31. If $y_0(t)$ is a solution of the initial value problem $y''(t) + y'(t) - 6y(t) = 0$; $y(0) = 7$ and $y'(0) = \lambda$, then determine the value of λ such that $\lim_{t \rightarrow \infty} y_0(t) = 0$.
32. Let g be a differentiable function of two real variables, and f be an analytic function of a complex variable z such that $f(z) = e^x \sin y + ig(x, y)$, $z = x + iy \in \mathbb{C}$. Then, what is the value of $g(3, 2) - g(1, 2)$?
33. Find the volume of the solid of revolution formed by rotating the region bounded by the curves $y = \sqrt{x}$ and $y = x^3$ about the X -axis.

34. Evaluate: $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} e^{\sqrt{1+t}} dt}{x^2} dx$.

35. Find the values of the constant a, b, c so that the directional derivative of scalar function $\phi(x, y, z) = ax^2 - by^2 + cz^2$ at the point $(1, 1, 2)$ has a maximum magnitude 4 in the direction parallel to Y -axis.

36. Examine the convergence of the improper integral $\int_0^1 \frac{\log(1/x)}{x^p} dx$ ($p > 0$).

37. Using Laplace transform, solve the initial value problem $y'' + 2y' + 2y = 0; y(0) = 0, y'(0) = 1$.

38. If $S_n = \sum_{k=1}^n \frac{1}{k}$, and $I_n = \int_1^n \frac{x - [x]}{x^2} dx$ for each $n \geq 1$, then determine the value of $S_{10} + I_{10}$.

Institute of Mathematics

Subject: Geology

Time: 2 Hours

Full Mark: 100

Answer ALL questions

1. Answer the following questions in one or more words. 2 × 30
- (i) Write the symmetry elements of class $\bar{3}2/m$.
 - (ii) Name the class in which talc crystallizes.
 - (iii) What is the amount of angle between positive end of a_2 and negative end of a_3 axes of hexagonal system?
 - (iv) Name the mineral in the hardness box which crystallizes in orthorhombic system.
 - (v) Name any two polymorphs of TiO_2 .
 - (vi) Name the mineral having four sets of cleavage.
 - (vii) Name the silicate structure of orthoclase.
 - (viii) Which is known as blue planet?
 - (ix) Name the deepest depth zone of ocean.
 - (x) Name the place in Odisha from which the oldest rock has been reported.
 - (xi) Basket of eggs topography refers to which geomorphic feature?
 - (xii) Name the seismic wave having maximum amplitude.
 - (xiii) A bed dips 30° due $S45^\circ E$; what is its dip amount in $N45^\circ E$?
 - (xiv) Name the type of fault in which hanging wall has moved up relative to the foot wall.
 - (xv) Name the fold in which both the limbs dip at equal angles in the same direction.
 - (xvi) Name the volcanic equivalent of diorite.
 - (xvii) Name the process by which natural glass is converted to crystalline material.
 - (xviii) Name the texture that results from intergrowth between quartz and orthoclase.
 - (xix) In grain size study of sediments, what is the value of 2Φ .
 - (xx) Which type of ripple mark is formed by the action of waves?
 - (xxi) Charnockite is formed in which facies of metamorphism?
 - (xxii) Name the process by which bauxite is formed.
 - (xxiii) Write the chemical composition of siderite.
 - (xxiv) Name the ore minerals used for extraction of iron in India.

